

An Empirical Study of Small Populations based on Taiwan's Data

Hsin-Chung Wang¹ Jack C. Yue²

Abstract

The estimation of mortality and disease incidence rates for small populations poses significant challenges for insurers in meeting regulatory compliance requirements, such as those imposed by the European prudential framework Solvency II. These difficulties primarily arise from data scarcity and the substantial volatility inherent in estimates derived from limited observations. To alleviate these issues, several methodologies have been proposed that effectively enlarge the underlying data set—for instance, through multi-population mortality models or the incorporation of reference populations—to reduce the stochastic variation in small-sample estimates. Such approaches, however, rest critically on the assumption that the reference population exhibits comparable demographic and epidemiological characteristics to those of the target population. A frequently adopted strategy involves pooling data from geographically adjacent districts; nevertheless, neighboring areas often differ markedly in their socioeconomic and demographic compositions, which may induce pronounced heterogeneity in age-specific mortality and incidence patterns.

In this study, we propose a data-driven framework for identifying suitable reference populations to enhance the goodness-of-fit of mortality estimation in small populations. Specifically, cluster analysis is employed to group small populations together with their neighboring districts, thereby constructing reference populations using techniques grounded in the heterogeneity measures proposed by Lee (2003). The study areas are partitioned into clusters exhibiting intra-group homogeneity and substantial inter-group heterogeneity in mortality rates. The proposed methodology is empirically validated using mortality data at the county and township levels in Taiwan. Based on the constructed reference populations, life tables are established, and life expectancy estimates for small populations are subsequently derived.

Keywords: Standardized Mortality Rate, Small Population, Population Projection, Age-Specific Rates, Health Inequality

¹ Professor, Department of Statistical Information and Actuarial Science, Aletheia University, New Taipei City, Taiwan, Republic of China

² Professor, Department of Statistics, National Chengchi University, Taipei, Taiwan, Republic of China

1. Introduction

In recent years, many countries have undergone rapid population aging, driven by persistently low fertility rates and continuous improvements in longevity. In addition, factors such as the “salmon bias” effect among the elderly, migration toward areas with better medical resources, and climate-related anomalies have further accentuated the demand for population projections at fine spatial scales, particularly at sub-county or sub-city (e.g., township) levels. A growing body of research has documented substantial and widening health disparities across social groups, geographic regions, and socioeconomic strata (Dwyer-Lindgren et al., 2016; Singh and Lin, 2019; Sanzenbacher et al., 2019; Benjamins et al., 2021). For instance, Baltrus et al. (2019) examined all-cause mortality in the United States from 1935 to 2014 and found a long-term decline that reversed after 2015, with the aggregate national trend masking considerable heterogeneity in local mortality trajectories. Similarly, Petrelli et al. (2024) investigated the association between socioeconomic status and avoidable mortality in Italy, revealing that regional responses to health policies varied substantially. Tetzlaff et al. (2024) likewise reported pronounced regional disparities in both mortality levels and temporal trends across the German federal states.

Regional disparities in health outcomes are also striking in Taiwan. According to the Taiwan Health Inequality Report¹, which analyzed township-level data (average population approximately 65,000) from 2001 to 2010, the life expectancy in the most socioeconomically deprived townships was roughly eight years shorter than that observed in the most affluent ones. Moreover, across nearly all levels of township deprivation, a clear regional gradient in life expectancy was evident: townships located in the eastern region exhibited the lowest longevity, whereas those in the northern region achieved the highest, even at comparable levels of socioeconomic disadvantage. Further disparities are observed when comparing life expectancy at birth among the six metropolitan areas, various counties, and Indigenous populations. Notably, female residents in Taipei City live, on average, up to twenty years longer than male Indigenous residents in mountainous areas. Beyond gender differences,

¹ Taiwan Health Inequality Report was published by the Health Promotion Administration, Ministry of Health and Welfare in 2017.

these findings strongly reinforce the patterns of health inequality documented in the national report (Figure 1).

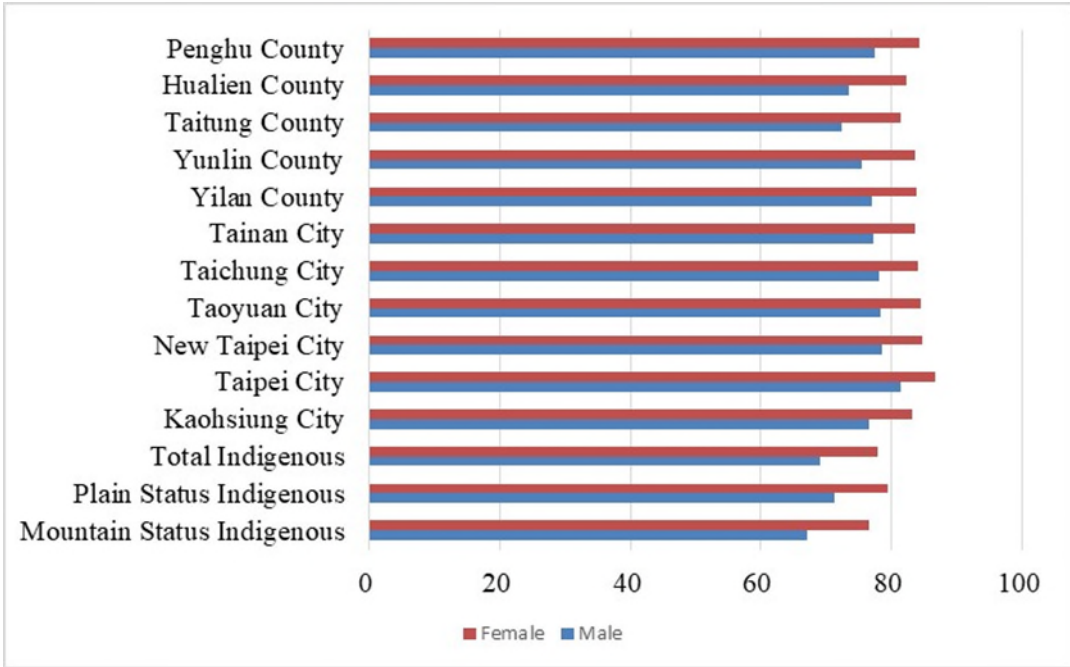


Figure 1. Life Expectancy at Birth (2017-19)

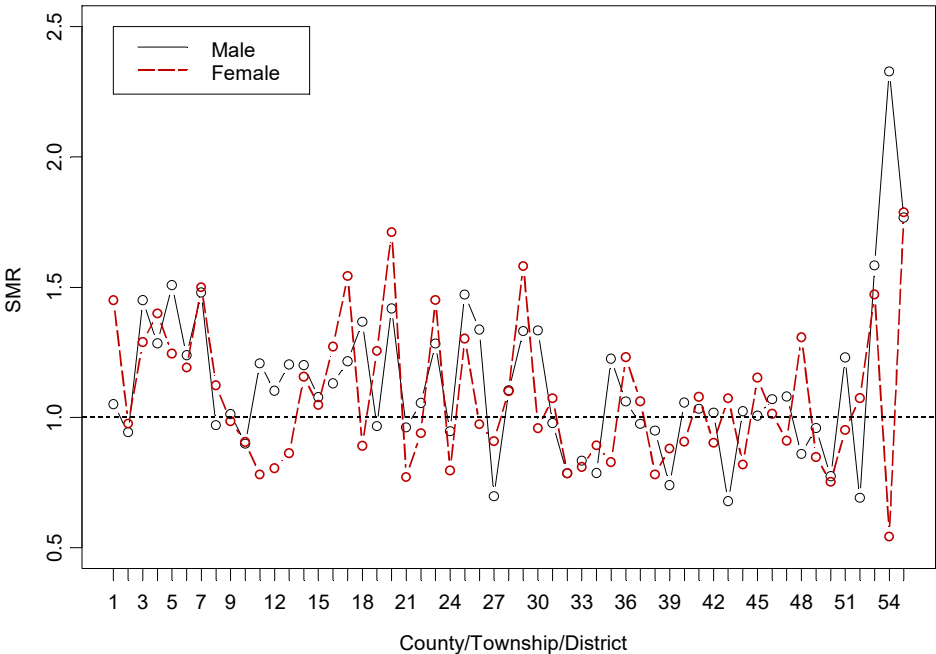


Figure 2. Standardized Mortality Ratio of 55 Townships in Taiwan (2017-19)

Regional disparities become even more pronounced at the township level. Figure 2 presents the standardized mortality ratios (SMRs) for 55 Indigenous townships in Taiwan—comprising 30 mountain Indigenous townships and 25 plain Indigenous townships—based on data from the Taiwan Indigenous Peoples Open Research Data (TIPD). The reference population in this analysis consists of all Indigenous people in Taiwan. An SMR below 1 indicates a mortality rate lower than that of the total Indigenous population. The results show that 19 and 27 townships exhibit SMR values below 1 for males and females, respectively, including 7 mountain and 12 plain Indigenous townships for males, and 13 mountain and 14 plain Indigenous townships for females. Approximately half of the Indigenous townships display higher mortality risks relative to the overall Indigenous population, suggesting that substantial health inequalities persist even within Indigenous communities, potentially attributable to differences in geographic accessibility, healthcare resources, and socioeconomic conditions.

Alexander et al. (2017) emphasized that obtaining reliable mortality estimates for small populations is essential for examining health inequalities within a country. However, producing accurate estimates of mortality or incidence rates for small populations remains challenging. Lee (2003) demonstrated that, at the county or township level, age-specific mortality rates can become highly unstable, and the number of deaths in certain age groups may even be zero, thereby posing difficulties for the application of popular mortality models such as the Lee–Carter model (Wang et al., 2018; Yue et al., 2021). In practice, when dealing with small-area populations—particularly among younger age groups—zero death counts are common. To mitigate this problem, most official life tables are constructed using three- or five-year aggregated data, followed by interpolation procedures to derive single-year age-specific mortality rates.

On the other hand, the accurate estimation of mortality and disease incidence rates, grounded in empirical evidence, is fundamental to ensuring the fair pricing and sound risk assessment of insurance products. Within the European prudential framework, Solvency II explicitly underscores the importance of reflecting actual experience in the calibration of mortality and morbidity assumptions

to fully capture potential risks. In practice, however, relevant empirical data are often limited, and information from products with superficially similar characteristics may not be directly transferable. For instance, the Taiwan Cancer Registry Annual Report documents substantial heterogeneity in age-specific incidence rates and peak ages across different cancer types. Comparable challenges also arise in the pricing of annuities, investment-linked policies, and deferred annuities, where factors such as consumer selection effects, small portfolio sizes, and the absence of claims data complicate model specification and parameter estimation—particularly when long-term projections are required.

Stochastic mortality models have become the standard framework for long-term mortality forecasting. Among the most influential are the Lee–Carter model (Lee and Carter, 1992) and its subsequent extensions (Brouhns et al., 2002; Booth et al., 2008; Renshaw and Haberman, 2006; Plat, 2009; Yang et al., 2010; Mitchell et al., 2013), as well as the Cairns–Blake–Dowd (CBD) model (Cairns et al., 2006). Most of these belong to the broader class of age–period–cohort (APC) models (Cairns et al., 2009), which have proven effective in capturing systematic mortality trends across large populations.

However, the direct application of such models to small populations often results in unstable parameter estimates. For example, Wang et al. (2018) demonstrated that fitting the Lee–Carter model to small-population data can lead to significant estimation bias (see also Booth et al., 2008; Jarner and Kryger, 2011). Similarly, Chen et al. (2017) found that the performance of the CBD model is highly sensitive to sample size, with substantial variability in the estimated parameters. To mitigate these issues, multi-population mortality models have been proposed (Li and Lee, 2005; Cairns et al., 2011; Dowd et al., 2011; Hyndman et al., 2013; Villegas et al., 2014; D’Amato et al., 2014; Li et al., 2015; Enchev et al., 2017; Eriksson and Christoffersen, 2020; Coulomb et al., 2020). These approaches jointly model small and reference populations, effectively borrowing strength from the latter to stabilize estimates and reduce sampling variability. Empirical evidence suggests that incorporating information from a reference population can substantially reduce both bias and volatility in mortality forecasts for small populations (Menzietti et al., 2019; Yue et al., 2021).

Previous studies have demonstrated that both multi-population models and the incorporation of information from reference populations can effectively alleviate the challenges of mortality estimation in small populations. These approaches can be viewed as methods of increasing the effective sample size and reducing estimation variability (Ahcan et al., 2014; Menzietti et al., 2019). However, there are no universally accepted guidelines regarding the selection of an appropriate reference population; such decisions are typically made on a case-by-case basis. A common practice is to combine historical data of the target small area with data from adjacent or demographically similar regions. Nevertheless, neighboring areas may exhibit distinct socioeconomic or demographic structures. For instance, Edwards and Tuljapurkar (2005) argued that mortality differentials across neighboring regions are not necessarily correlated with socioeconomic similarity. In light of this, the present study adopts regional mortality homogeneity over time as the principal criterion for selecting reference populations. By integrating mortality information from reference areas that exhibit consistent temporal patterns with the target small population, the study aims to enlarge the effective population base and enhance the reliability of mortality forecasts for small areas.

The remainder of this paper is organized as follows. Section 2 introduces the research methodology and the mortality measures employed in this study. Section 3 presents the exploratory data analysis based on mortality data from 368 townships in Taiwan. In Section 4, the proposed approach is implemented using machine learning techniques to cluster Taiwan's townships and identify suitable reference populations for small areas, based on the mortality heterogeneity index. Information from these reference populations is then utilized to construct life tables and estimate life expectancy at the township level. Finally, Section 5 discusses the main findings and potential applications of the proposed framework to life insurance products.

2. Methodology

In this study, we examine mortality characteristics across 368 townships in Taiwan using three indicators: the Crude Death Rate (CDR), the Standardized Death Rate (SDR), and the Standardized Mortality Ratio (SMR). To reduce dimensionality and extract common underlying patterns, we apply

Principal Component Analysis (PCA) and interpret the principal components through their associated eigenvectors. We further classify the townships into distinct clusters using the K-means algorithm.

To address small-area estimation challenges, we adopt the heterogeneity index proposed by Lee (2003) to quantify similarity between a target small population and a selected reference population. We then combine partial SMR (PSMR) with the Whittaker ratio smoothing method to improve mortality prediction for small areas. The indicators, clustering procedure, and smoothing methods are introduced in the following sections.

I. Crude Death Rate (CDR)

$$CDR = \text{Total Resident Deaths} / \text{Total population}.$$

The CDR is defined as the total number of resident deaths in a specified geographic unit (e.g., city, district, county, or township) divided by its total population.

II. Standardized Death Rate (SDR)

$$SDR = \frac{\sum (m_x \times P_x^S)}{\sum P_x^S}, \quad (1)$$

where m_x is an age-specific death rate of study population for age group x , P_x^S is number of persons of standard population for age group x . The SDR is a mortality indicator that is adjusted for differences in age structure among different populations, thus allowing for a fair comparison of mortality levels in different regions or at different times.

III. Standardized Mortality Ratio (SMR)

$$SMR = \frac{\sum_x d_x}{\sum_x e_x} = \frac{\sum_x d_x}{\sum_x n_x \cdot q_x} \quad (2)$$

where d_x and e_x are the observed and expected numbers of deaths at age x for the small population, n_x is the population size of age x for the small population, and q_x is the central death (or mortality) rate of age x for the reference (or large) population. The SMR compares the observed number of deaths in a specific small population with the expected number of deaths based on the mortality experience of a reference population. An SMR greater (less) than 1 indicates that the small population exhibits higher (lower) overall mortality relative to the reference standard.

IV. The partial SMR (PSMR)

The PSMR incorporates supplementary information from larger populations to mitigate estimation bias and improve robustness when the observed population is sparse or unstable. The partial SMR of Lee (2003) is formulated under the assumption that the graduated mortality rates satisfy

$$v_x = u_x \times \exp\left(\frac{d_x \times \hat{h}^2 \times \log(d_x/e_x) + (1 - d_x/\sum d_x) \times \log(\text{SMR})}{d_x \times \hat{h}^2 + (1 - d_x/\sum d_x)}\right) \quad (3)$$

where u_x and v_x denote the observed and graduated mortality rates at age x , respectively. Similarly, d_x and e_x represent the observed and expected numbers of deaths at age x for the small population. The parameter $\hat{h}^2$ is then estimated to quantify the heterogeneity in mortality rates between the small population and the reference (larger) population, and it is defined as

$$\hat{h}^2 = \max\left(\frac{\sum((d_x - e_x \times \text{SMR})^2 - \sum d_x)}{\text{SMR}^2 \times \sum e_x^2}, 0\right) \quad (4)$$

A larger value of $\hat{h}^2$ indicates greater differences in age-specific mortality rates between the small population and the reference population, reflecting higher mortality heterogeneity or greater dissimilarity in the shape of the age-specific mortality curves. When $\hat{h}^2 = 0$, no rate-ratio heterogeneity is detected. We employ this measure as a criterion for selecting an appropriate reference population.

V. Whittaker ratio

The Whittaker graduation method (London, 1985) seeks to minimize the following objective function:

$$M = F + hS = \sum_{x=1}^n w_x (v_x - u_x)^2 + h \sum_{x=1}^{n-z} (\Delta^z v_x)^2, \quad (5)$$

where u_x and v_x denote the observed and graduated mortality rates (or ratios) at age x , respectively; w_x is the weight assigned to age x ; n is the total number of ages (or age groups) considered; and h and z are smoothing parameters to be determined. In this study, we set h equal to the average population size of a single age or 5-year age group, and set z to be same as in our previous studies (Wang et al., 2012).

The Whittaker ratio graduation method is an extension of the traditional Whittaker approach. Instead of smoothing mortality rates, we apply the method to the mortality ratios between the small population and its reference population. Specifically, the observed mortality rate u_x is replaced by the

u_x^r observed mortality ratio at age x in the objective function. In doing so, the graduation is performed directly on the relative mortality structure, which is more suitable when incorporating information from a reference population in small-area mortality estimation, i.e.,

$$u_x^r = \frac{q_x^s}{q_x^R} \quad (6)$$

where q_x^s is the mortality rate of the small population, q_x^R is the mortality rate of the reference population. Accordingly, the Whittaker ratio graduation method seeks to find v_x^r by minimizing

$$M^r = \sum_x w_x (v_x^r - u_x^r)^2 + h \sum_x (\Delta^z u_x^r)^2 \quad (7)$$

After smoothing the ratios, the graduated mortality rate for the small population is obtained as $v_x = v_x^r \times q_x^R$.

Small-area populations are often subject to substantial random fluctuations in observed mortality rates due to limited population size. Whether the research focus is on age-specific mortality or life expectancy, the variance of mortality-based estimators is generally inversely proportional to population size. Therefore, when the exposed population is insufficient, graduation methods are required to reduce such volatility. In principle, graduation should not distort the underlying information contained in the observed mortality data, such as mortality trends or life expectancy patterns. Its primary objective is to reduce random fluctuations in the observed mortality rates, thereby enabling researchers to better identify the overall structure of the mortality profile.

In mortality graduation, smoothness and goodness-of-fit represent related but distinct objectives. Increased smoothness may reduce irregular random fluctuations in sparse mortality data, but excessive smoothing can also decrease adherence to the observed mortality rates. Accordingly, the proposed framework does not interpret smoother mortality schedules as universally superior. Instead, the objective is to achieve a practical balance between stability, interpretability, and consistency with the observed age-specific mortality patterns. In other words, the smoothing parameter was selected to achieve a practical balance between reduction of irregular fluctuations and preservation of the main age-specific mortality structure, rather than through formal optimization of predictive fit.

Accordingly, when evaluating different small-area mortality estimation methods, in addition to

visually examining the mortality patterns before and after graduation, we adopt the degree of fluctuation as one of the key performance metrics. For instance, in the Whittaker graduation method, the second term of the objective function penalizes excessive variation between adjacent age-specific mortality rates (i.e., smoothness), defined as

$$\sum_{x=1}^{n-z} (\Delta^z v_x)^2 \quad (8)$$

where z is typically set to 3, although values of 1 or 2 are sometimes used in practice. Moreover, because the number of older individuals is relatively small in many small areas, the assessment of smoothness in this study focuses on ages 0–69.

Note that, although the heterogeneity index is not a likelihood-based goodness-of-fit statistic, it can still be interpreted as a structured deviation measure that quantifies discrepancies between regional age-specific mortality patterns and their corresponding reference-group mortality schedules. In this sense, smaller heterogeneity index values indicate closer agreement between the observed and reference mortality profiles, making the index conceptually analogous to deviation-based goodness-of-fit measures such as the mean squared discrepancy.

In addition, hierarchical clustering was initially employed as an exploratory tool to identify plausible numbers of clusters and to visualize the overall similarity structure among regions. Conditional on the candidate values of K , k -means clustering was subsequently applied to obtain stable and reproducible partitions. This two-stage strategy balances interpretability and robustness while avoiding reliance on a single clustering method.

3. Exploratory data analysis (EDA)

In this study, we analyze annual single-age population data for 22 counties/cities and 368 townships/districts in Taiwan from 1997 to 2019, supplemented by corresponding single-age death counts from 2007 to 2018. Based on these data, we compute three mortality indicators—CDR, SDR, and SMR—to classify mortality characteristics across the townships/districts. The SDR is calculated using the WHO 2000 world standard population as the standard age distribution, while the SMR is computed by adopting Taiwan’s total population as the reference population over the study period.

Taiwan's administrative divisions are primarily used to delineate government power, electoral representation, and financial jurisdiction. Despite its relatively small land area (36,006 km²), Taiwan features substantial geographical heterogeneity, spanning plains, hills, and mountainous regions. Consequently, even townships within the same municipality may be situated in distinctly different terrain, potentially leading to variation in population structure and health outcomes.

To illustrate these disparities, we examined the Crude Death Rate (CDR) by township during 2016–2018. In Shiding District—a mountainous township in New Taipei City—the male CDR was 0.0129, nearly 1.8 times that of the urbanized Banqiao District (0.00691) within the same municipality. This suggests that simply using the entire New Taipei City as the reference population for mortality estimation may introduce systematic bias. Moreover, the regional contrast is even more pronounced nationwide: the male CDR in Changbin Township of Taitung County (eastern region) reached 0.0214, approximately 5.5 times higher than that in Luzhu District of Taoyuan City (western region). Similarly, the female CDR in Changbin Township was 6.8 times higher than in Nangan Township, Lianjiang County (outlying islands), underscoring substantial spatial disparities (Figure 3).

Given that the CDR is sensitive to differences in population age structure and thus provides a poor basis for comparing mortality risks across regions (Newell, 1988), the SMR is widely used in public health research to enable fair comparisons between different geographic areas. Anson (2018) demonstrated that SMR offers a more reliable indicator of local mortality levels and their association with socioeconomic conditions. Likewise, Orwat-Acedańska (2024) utilized SMR-based methods to examine small-area mortality disparities in Poland, further confirming the applicability of SMR in regional mortality assessment.

Following this approach, we computed the SMR by treating Taiwan's total population as the reference and each township as the comparison unit. Figure 4 illustrates that 244 townships exhibit male mortality levels above the national average, while 124 townships fall below it—mainly concentrated in the western plains and certain outlying islands. For females, 204 townships exceed the national average and 164 remain below, consistent with the lower overall female mortality relative to males. In general, elevated mortality is observed in townships located in central and

eastern mountainous areas as well as parts of southern Taiwan. To further control for differences in population age and gender structure inherent in the CDR, we examined the SDR across townships (Figure 5). The resulting spatial patterns closely resemble those observed in the SMR analysis, reinforcing the presence of distinct regional mortality disparities in Taiwan.

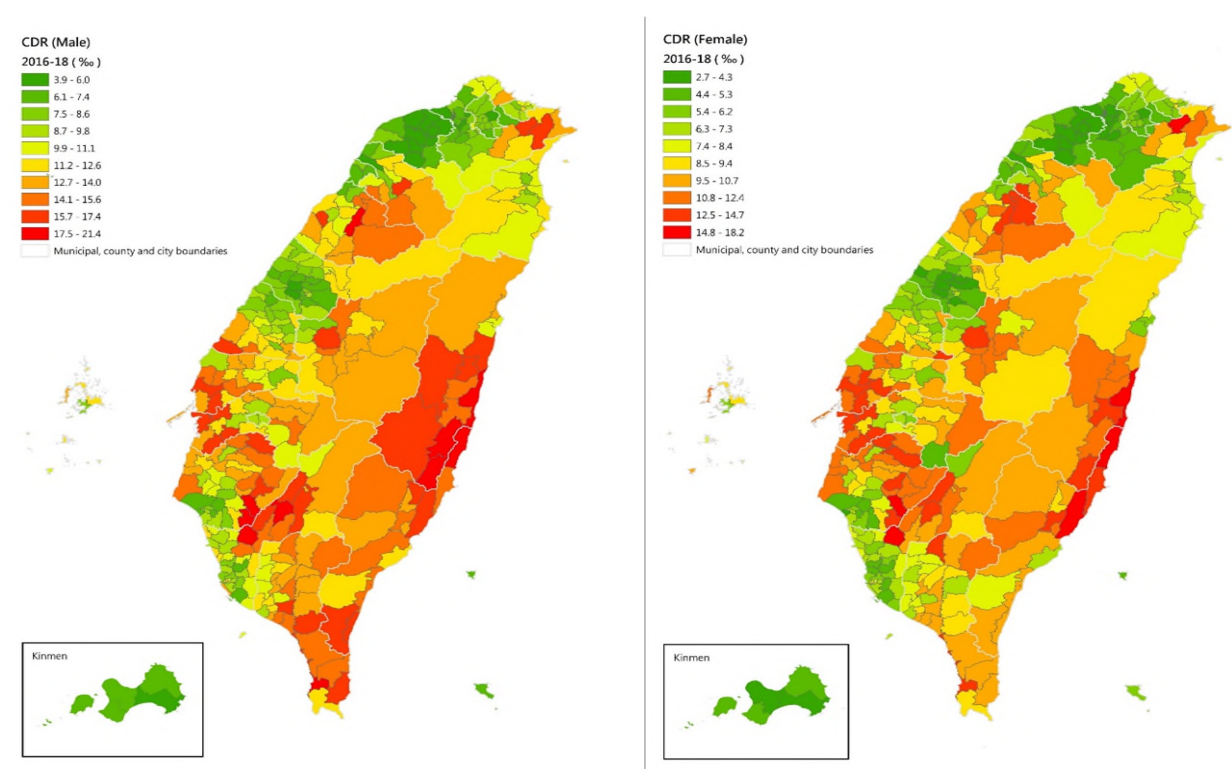


Figure 3. CDR heatmap of 368 townships/districts in Taiwan

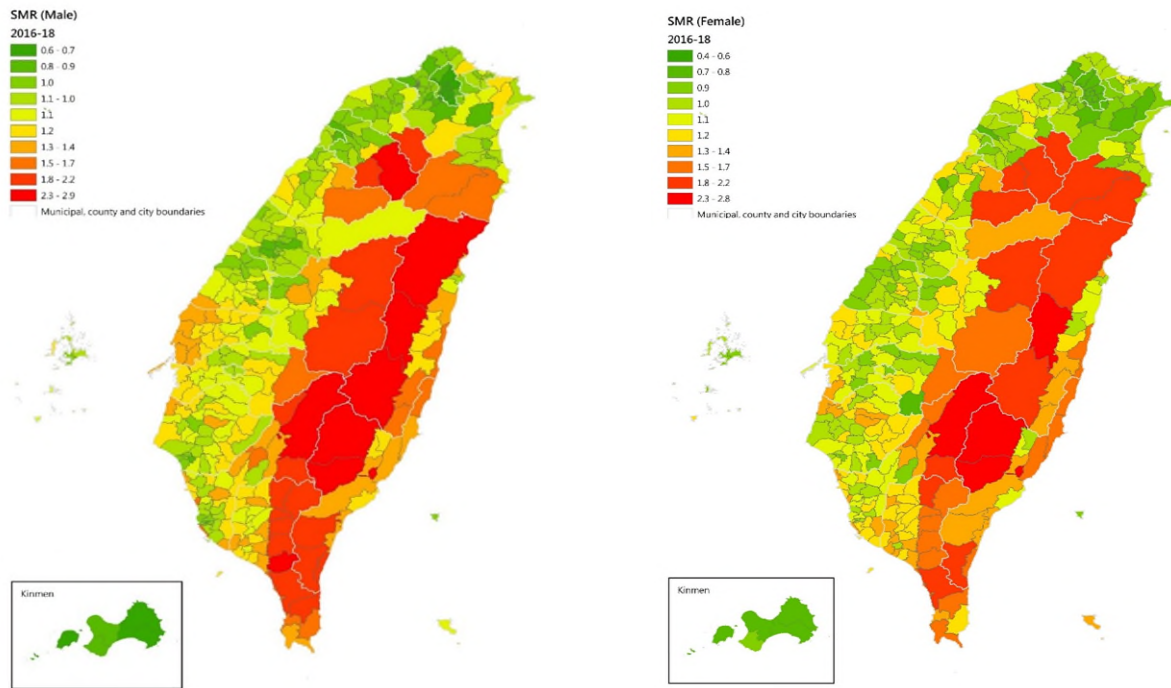


Figure 4. SMR heatmap of 368 townships/districts in Taiwan

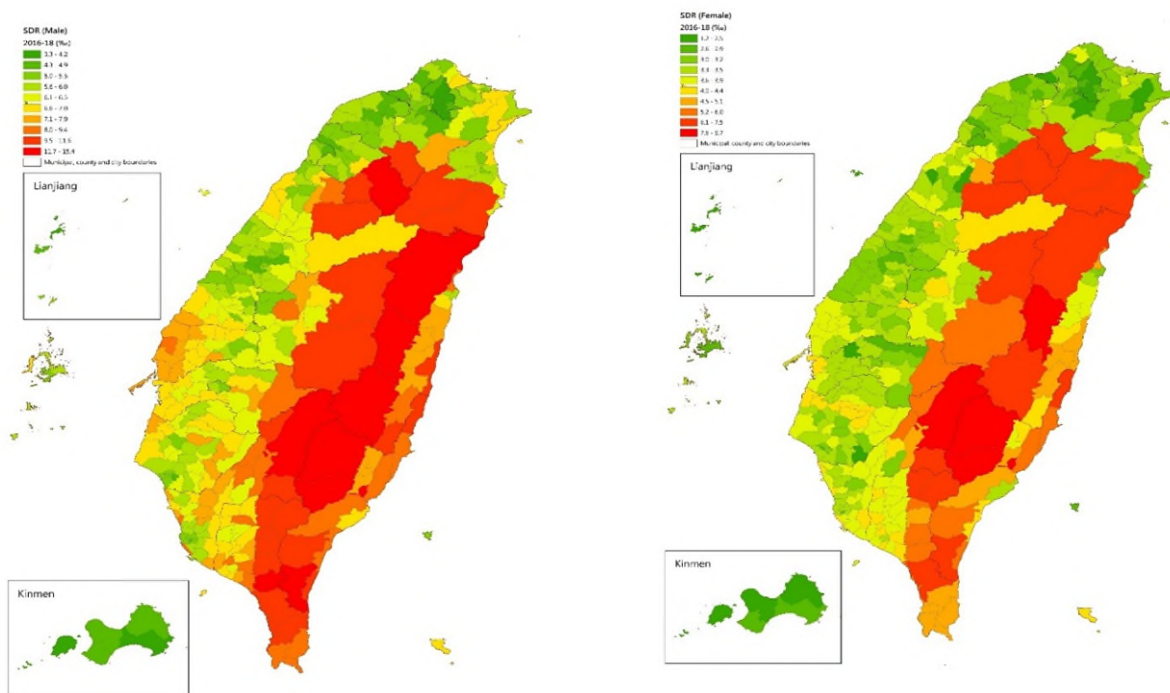


Figure 5. SDR heatmap of 368 townships/districts in Taiwan

We examined the population distribution of Taiwan's townships/districts using the average total population (both sexes combined) from 2016 to 2018 (Figure 6). The results indicate substantial heterogeneity in population size across administrative units. The smallest township/district had only

676 residents, whereas the largest had 554,373. In total, 29 townships/districts had populations exceeding 200,000; 40 were between 100,000 and 200,000; 58 between 50,000 and 100,000; 177 between 10,000 and 50,000; and 64 had fewer than 10,000 residents. Notably, approximately 65% of Taiwan's 368 townships/districts have populations below 50,000, yet they represent only 21% of the national population. These disparities underscore the high degree of urban concentration and the challenges associated with mortality estimation in sparsely populated areas, where fluctuations in observed mortality rates are more likely.

To further illustrate this issue, we selected representative township pairs from three major municipalities—Songshan and Wanhua Districts in Taipei City; Banqiao and Shiding Districts in New Taipei City; and Zhongli and Fuxing Districts in Taoyuan City. Examination of the empirical male age-specific mortality rates (Figure 7) reveals that considerable volatility persists even within metropolitan regions. Additionally, some districts, such as Wanhua, Shiding, and Fuxing, exhibit missing observations in several age groups due to small population sizes, highlighting the need for robust estimation methods in small-area mortality modeling.

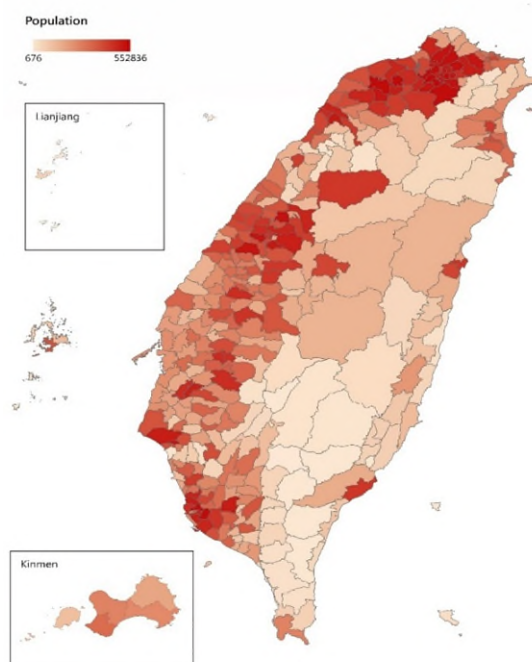


Figure 6. Population distribution heatmap of Taiwan's 368 townships and districts

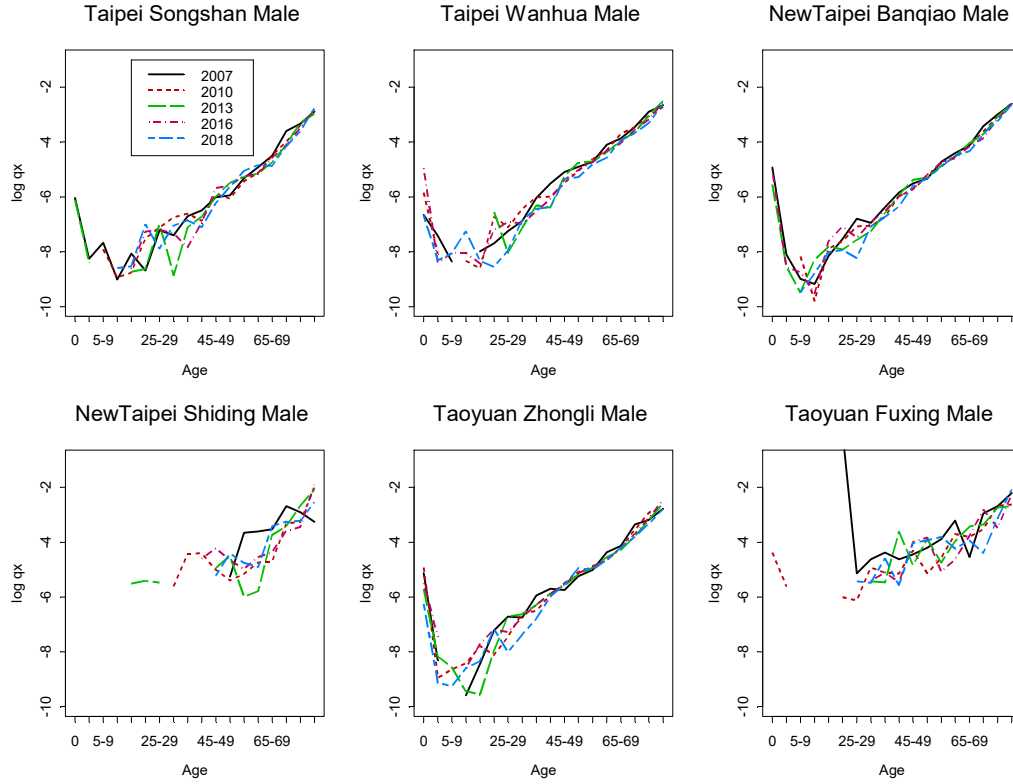


Figure 7. Regional male age-specific mortality rate (2007-2018).

These findings indicate that substantial intra-regional disparities in mortality exist even among townships located within the same metropolitan living circles, particularly when geographical characteristics differ. Therefore, expanding the effective sample size for small populations requires the use of reference populations that exhibit sufficiently homogeneous mortality characteristics, in order to mitigate estimation bias. In the subsequent section, we introduce a clustering-based approach for identifying suitable reference populations and describe the application of smoothing techniques to address missing or volatile age-specific mortality observations in small areas, thereby enhancing the reliability of mortality estimation.

4. Classification

As noted previously, the clustering patterns of mortality based on the CDR differ substantially

from those obtained using standardized indicators such as the SDR and SMR. Therefore, SDR and SMR were adopted as the primary measures in subsequent analyses. The study utilized mortality data for 368 townships/districts in Taiwan from 2007 to 2018, categorized into five age groups (0, 1–4, ..., 100+). Given the diverse determinants influencing mortality across geographic regions, PCA was first conducted to quantify the variance explained by each component and the associated cumulative contribution over time, thereby informing the grouping strategy. Figure 8 illustrates the results for male SDR as an example. Retaining 3, 5, and 7 principal components explains 67%, 79%, and 88% of the total variance, respectively. To capture both temporal patterns and heterogeneity in mortality risk, K-means clustering was subsequently applied to the principal component scores. The optimal number of clusters was evaluated using the Elbow Method, Silhouette Score, and Gap Statistic.

However, the three-cluster configuration exhibited considerable within-group variation, likely attributable to Taiwan's diverse topography—where a single township may span plains, hills, and mountainous terrain—as well as substantial disparities in urbanization and healthcare accessibility. To enhance within-cluster homogeneity, Hierarchical Clustering was further employed, which indicated that five- or seven-cluster solutions were more appropriate. Accordingly, three clustering schemes—3-Group, 5-Group, and 7-Group—were established using the K-means method, providing clearer inter-group differentiation and improved internal consistency. Ward's minimum variance method was utilized as the hierarchical algorithm, based on squared Euclidean distances as the dissimilarity measure (Haraty et al., 2015). To compare grouping consistency, three-cluster solutions derived from SDR and SMR were evaluated. The results indicated a high degree of agreement. As reported in Table 1, for males, 27 townships/districts were consistently classified into the same group under both SMR- and SDR-based clustering, while four additional townships/districts were included in this group when using SDR, resulting in a total of 31. For females, the two indicators demonstrated a high degree of concordance, with 312 townships/districts assigned to the same group under both SMR and SDR classifications.

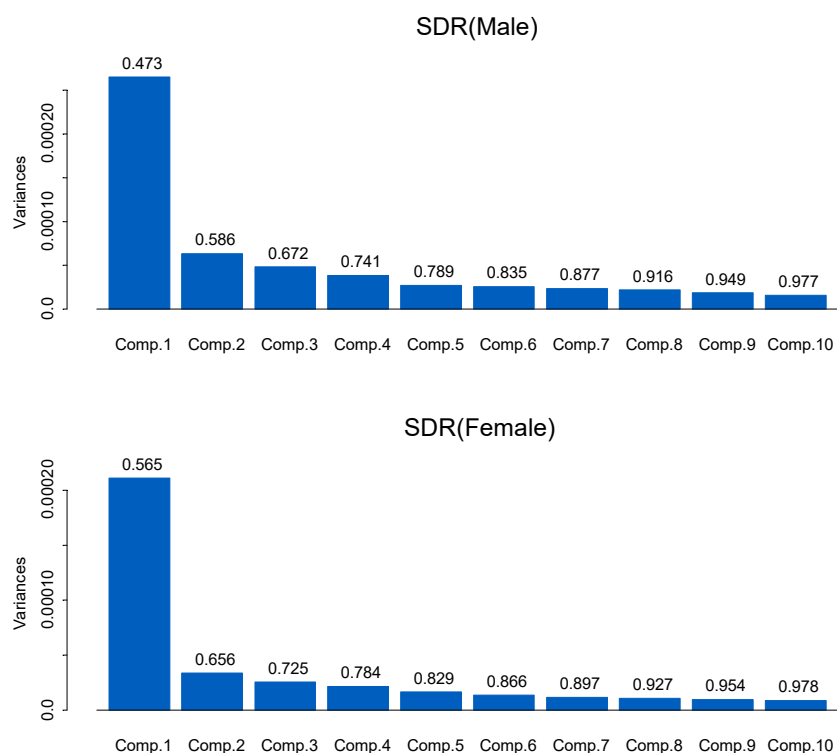


Figure 8. Screenshot- Cumulative Proportion of Explained Variance (SDR)

Table 1. Township/District Group Counts: SDR vs. SMR

3 Groups		SMR	SDR	Same Classification
Male	Group 1	27	31	27
	Group 2	191	190	183
	Group 3	150	147	139
Female	Group 1	312	333	312
	Group 2	44	34	23
	Group 3	12	1	1

Next, we evaluated within-cluster homogeneity in age-specific mortality patterns to assess the appropriateness of the three-cluster classification. To address this concern, we calculated township-level male SMRs for five age groups (0–14, 15–44, 45–64, 65–74, and 75+) using averages over the 2007–2018 period. We then evaluated within-cluster variation under the tri-cluster model by computing the coefficient of variation (CV) of the SMRs within each cluster, as presented in Table 2.

The results showed relatively high within-cluster variability in younger age groups, particularly the 0–14 age group, reflecting lower mortality rates and greater random fluctuations in smaller groups. However, the CV declined substantially with increasing age across all clusters, especially in the middle-aged and older age groups. In particular, the CVs for the 65–74 and 75+ age groups remained consistently low, indicating strong reasonably homogeneous age-specific patterns in the age ranges most relevant to mortality graduation and life expectancy estimation. These findings suggest that the proposed clustering approach is capable of capturing meaningful similarities in age-specific mortality structures, particularly at older ages where mortality patterns are more stable and informative.

Table 2. Within-cluster CV under the Three-Cluster Model for Males

Group	N	Mean Overall SMR	CV (0–14)	CV (15–44)	CV (45–64)	CV (65–74)	CV (75+)	CV (All Age)
1	27	2.24	0.79	0.26	0.18	0.20	0.11	0.18
2	191	0.98	0.50	0.30	0.17	0.12	0.10	0.11
3	150	1.25	0.46	0.32	0.17	0.12	0.07	0.10

In K-means clustering, the parameter k denotes the predefined number of clusters to which the observations are allocated. When the 368 townships/districts were classified into 3-, 5-, and 7-cluster structures, the spatial distributions and corresponding cluster sizes were obtained as illustrated in Figure 9 and summarized in Table 3. These results demonstrate that relying solely on administrative boundaries as reference populations for small-area mortality analyses is inadequate, as such an approach may mask substantial heterogeneity in mortality risk across different townships/districts.

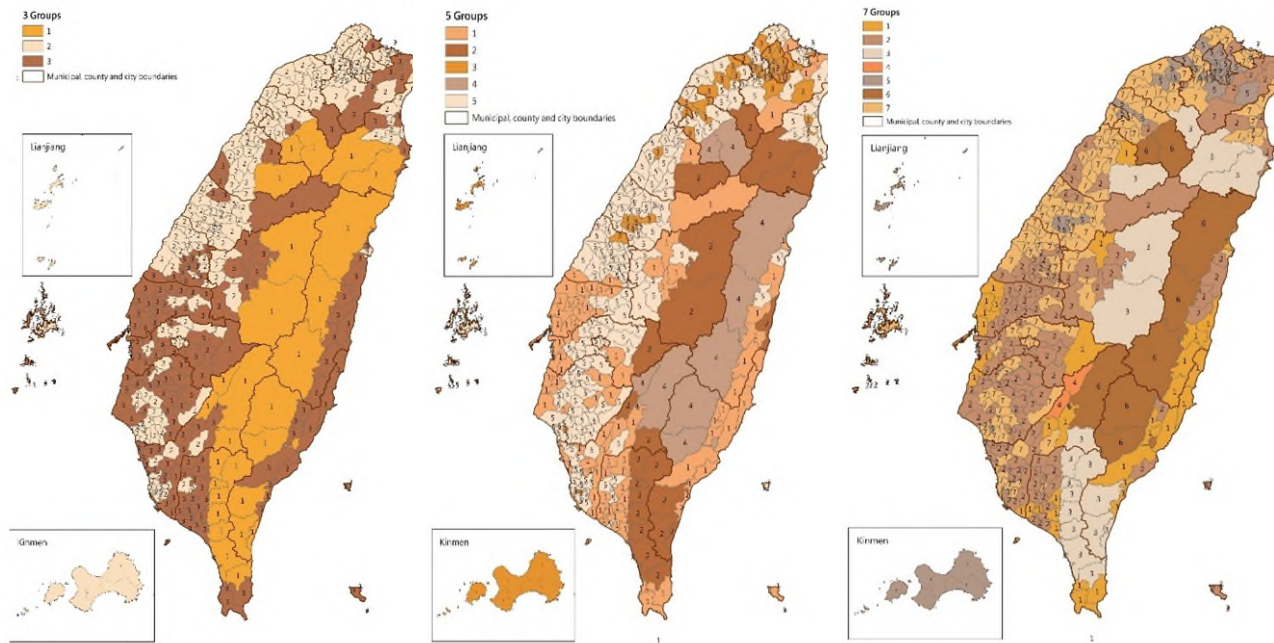


Figure 9. Mortality heatmaps of 3, 5, and 7 clusters (Male-SMR)

Table 3. Number of observations under 3, 5, and 7 clusters (SMR)

	3-Groups		5-Groups		7-Groups	
	Male	Female	Male	Female	Male	Female
Group 1	27	312	92	35	36	21
Group 2	191	44	22	11	121	1
Group 3	150	12	56	120	19	43
Group 4			9	2	2	11
Group 5			189	200	43	17
Group 6					8	130
Group 7					139	145

Given that a township may be allocated to different cluster solutions, each township thus possesses four candidate reference populations: (1) the administrative district to which it belongs (county or city), (2) the 3-Group classification, (3) the 5-Group classification, and (4) the 7-Group classification. To determine the most appropriate reference population for each township, Lee's (2003) heterogeneity index (h^2) was employed as the selection criterion. Larger values of h^2 indicate greater heterogeneity in mortality between the small population and the reference population, reflecting

substantial differences in the shape of the respective age-specific mortality curves. Conversely, smaller values correspond to higher similarity between the two curves, denoting lower heterogeneity. As an illustration, eight townships/districts were examined in detail. Wanhua District in Taipei City exhibited the lowest heterogeneity value within the 5-Group classification, leading to the selection of the corresponding reference population from Group 5 of the 5-Group solution (Table 4).

Table 4. Heterogeneity index values and reference population selection in 8 townships.

	Male Population(2016- 18)	Heterogeneity(2007-18 Average)				Reference Population	
		County/ City	3 Groups	5 Groups	7Groups		
Taipei Songshan District	96,327	0.007299	0.017499	0.011605	0.010606	Taipei City/ 7 groups	Group 5
Taipei Wanhua District	92,944	0.015735	0.008619	0.007423	0.007975	5 groups	Group 5
NewTaipei Banqiao District	270,217	0.081269	0.004348	0.003638	0.003886	5 Groups	Group 3
NewTaipei Shiding District	4,204	0.044059	0.027977	0.032203	0.028552	3 groups	Group 3
Taoyuan Zhongli District	202,412	0.005510	0.006156	0.007841	0.008433	3 groups/Taoyuan City	Group 2
Taoyuan Fuxing District	6,443	0.107205	0.066984	0.025733	0.026174	5 groups	Group 2
Taichung Xitun District	109,679	0.007209	0.007465	0.005552	0.005419	7 groups	Group 5
Taichung Heping District	5,882	0.061363	0.040106	0.036879	0.043127	5 groups	Group 1

Then, we applied two graduation methods (Whittaker ratio and PSMR) to construct the life tables of each township. Figure 10 depicts the mortality curve (red solid line) for Songshan District, Taipei City. Regardless of whether the three-year average (2016-2018) or the 2018 data is used, the goodness-of-fit of estimating age-specific mortality rates using Taipei City as the reference population is comparable to that achieved with the 7-Groups classification. Both the Whittaker ratio method and the PSMR demonstrate similar levels of goodness-of-fit. The goodness-of-fit of estimating age-specific mortality in Shiding District, New Taipei City (red solid line) using Taipei City as the reference population is poorer than that obtained using the 3-Groups classification. The Whittaker ratio method demonstrates superior goodness-of-fit compared to PSMR (Figure 11). Figure 12 reveals that in 2018, Wanhua District exhibited a higher baseline mortality rate compared to both Taipei City and Songshan District. In Shiding District, New Taipei City, data on individuals under 50 years of age is unavailable. However, the mortality rate among those aged 50 and above surpasses that of both New Taipei City as a whole and Banqiao District within New Taipei City. Following graduation using either the Whittaker ratio method or the PSMR, we observe that the graduated mortality rates in Wanhua District, Taipei City, exceed those observed in both Taipei City's abridged life table and Songshan District in 2018. Compared to both the abridged life table and Banqiao District in New Taipei City in 2018, Shiding District, New Taipei City exhibits the highest mortality rate (Figure 13). Our findings demonstrate that the graduated mortality curve, based on the chosen suitable reference population, exhibits strong consistency with the original mortality trend.

Finally, we selected men from two districts in each of the six cities as a sample to construct a life table, presenting life expectancy at birth. This was then compared with the abridged life table of the six cities published by the Ministry of Interior in 2018 (Table 5). For example, the life expectancy at birth in Taipei City's Wanhua District is 78.34 years, which is lower than both the Taipei City abridged life table (80.93 years) and Taipei City's Songshan District (80.2 years). This aligns with the original mortality trend. In Taoyuan City, the life expectancy at birth in Zhongli District is 79.43 years, which is higher than the city's abridged life table value of 77.86 years. However, the Fuxing

District in the same city has a significantly lower life expectancy of just 68.36 years. Similarly, in Kaohsiung City, the life expectancy at birth in Xinxing District is 79.69 years, exceeding the abridged life table for the city, which stands at 76.13 years. In contrast, the Maolin District in Kaohsiung has a much lower life expectancy of only 60.27 years.

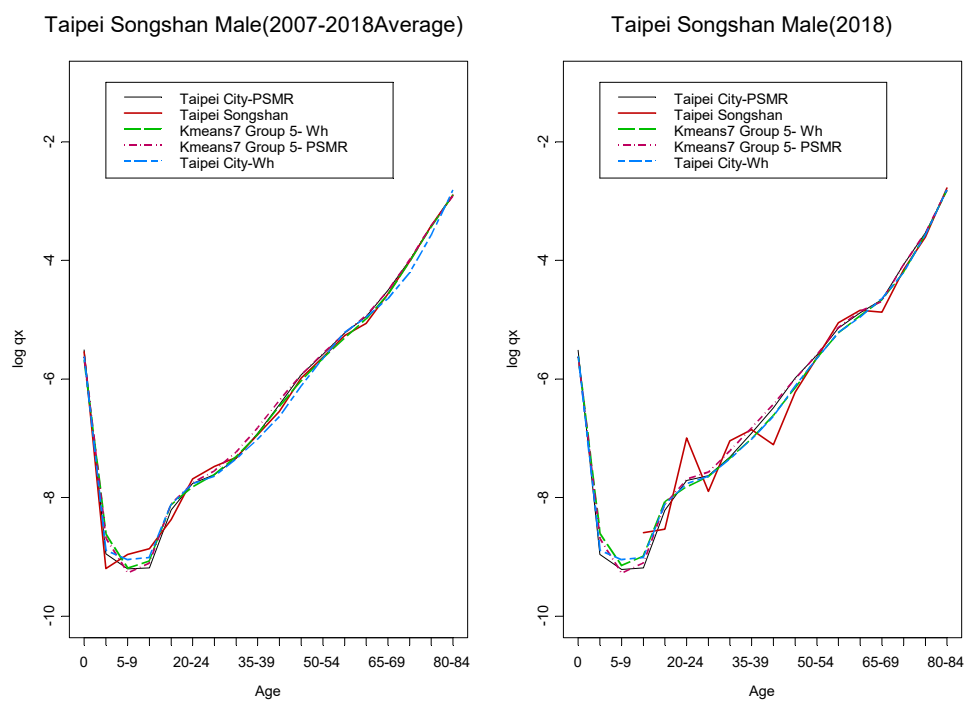
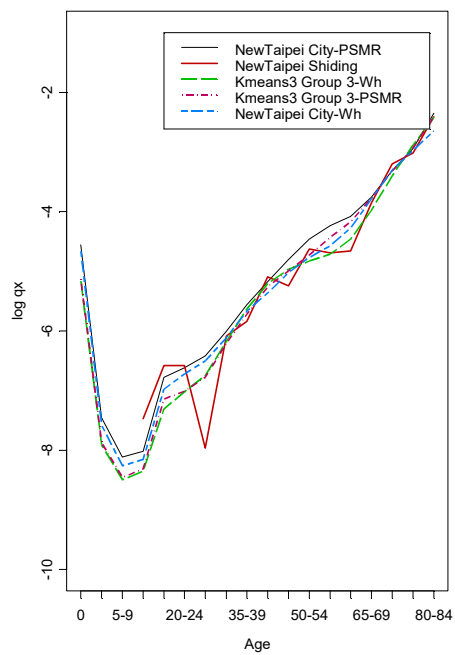


Figure 10. Mortality curve in Songshan District, Taipei City

NewTaipei Shiding Male(2007-2018Average)



NewTaipei Shiding Male(2018)

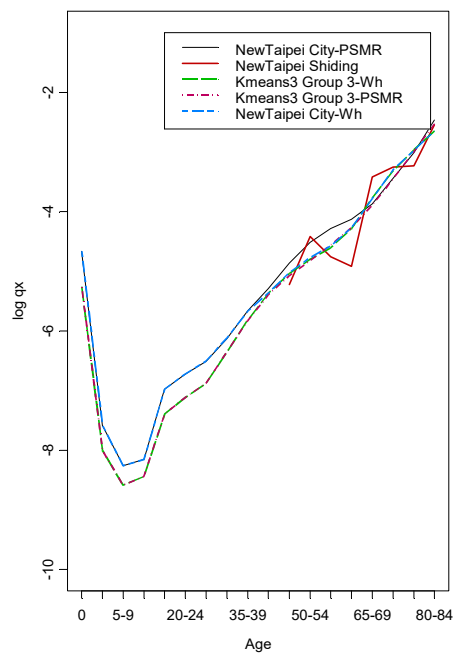
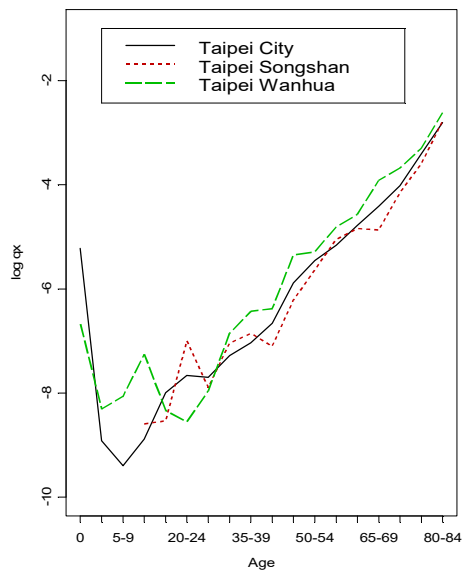


Figure 11. Mortality curve in Shiding District, New Taipei City

Male Original Mortality Rate(2018)



Male Original Mortality Rate(2018)

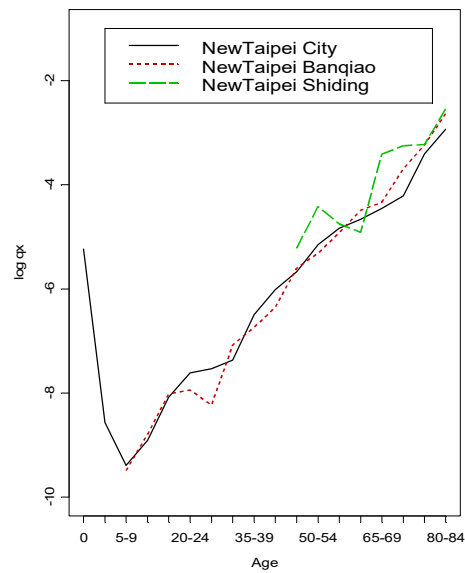


Figure 12. Raw mortality curve comparison (2018-Male)

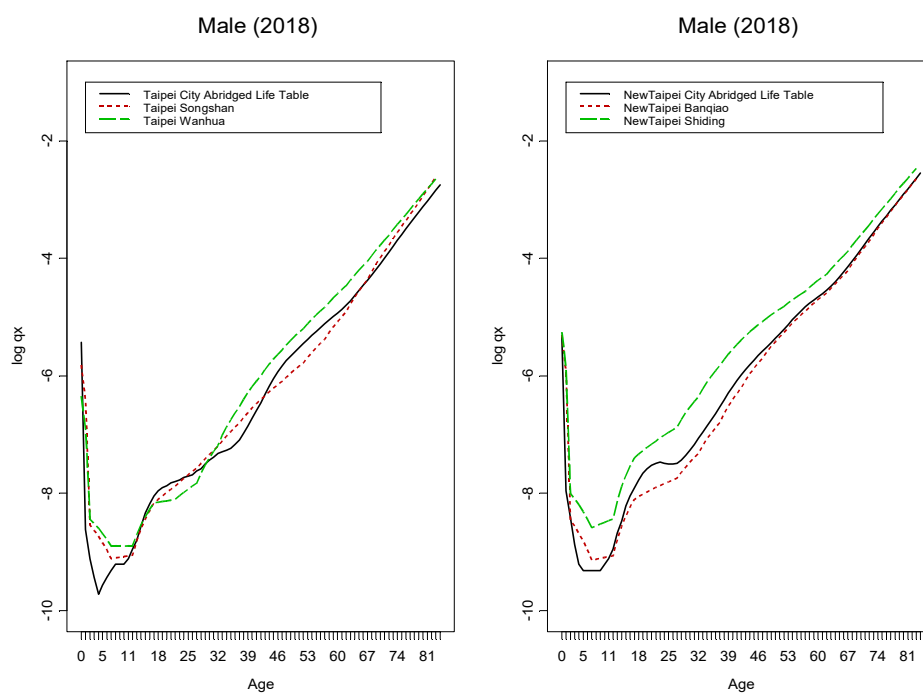


Figure 13. Comparison of mortality curves after graduation (2018-Male)

Table 5. Life expectancy at birth in six cities

	Life expectancy at Birth
Taipei City 2018 abridged life table	80.93
Taipei Songshan District	80.20
Taipei Wanhua District	78.34
NewTaipei City 2018 abridged life table	78.33
NewTaipei Banqiao District	78.72
NewTaipei Shiding District	74.30
Taoyuan City 2018 abridged life table	77.86
Taoyuan Zhongli District	79.43
Taoyuan Fuxing District	68.36
Taichung City 2018 abridged life table	77.61
Taichung Xitun District	79.00
Taichung Heping District	75.61
Tainan City 2018 abridged life table	76.79
Tainan Anping District	76.61
Tainan Beimen District	70.12
Kaohsiung City 2018 abridged life table	76.13
Kaohsiung Xinxing District	79.69
Kaohsiung Maolin District	60.27

We also assessed the performance of different small-area mortality estimation methods using smoothness as an evaluation criterion (Equation (8)). As an illustrative example, we considered male mortality in Wanhua District and Shiding District. For each method, we computed the sums of squared first-, second-, and third-order differences of age-specific mortality rates from ages 0 to 69. To enable comparison across approaches, we derived a smoothness ratio by dividing the smoothness measure of each estimated mortality schedule by that of the observed mortality rates. A ratio below one indicates that the estimated mortality curve is smoother than the observed counterpart, with smaller values reflecting reduced fluctuations between adjacent ages. Table 6 shows the smoothness ratios ($z = 1, 2, 3$) for the two types of reference populations, where G denotes the clustering-based reference population proposed in this study, and C denotes the reference population defined by the administrative jurisdiction (county/city) of the small area. The deviation between the estimated and

observed mortality rates increases with the order of differencing. The differences between the two types of reference populations are relatively small overall. The clustering-based reference population performs slightly better than the county/city reference population. In addition, the PSMR and Whittaker approaches each exhibit their own advantages.

Table 6. Smoothness Ratio of Wanhua and Shiding Districts

Township	Methods	Ages 0-69		
		$z = 1$	$z = 2$	$z = 3$
Wanhua	PSMR(G)	0.96	0.49	0.23
	Whittaker(G)	0.96	0.34	0.15
	PSMR(C)	1.08	0.68	0.38
	Whittaker(C)	0.99	0.40	0.15
Shiding	PSMR(G)	0.33	0.10	0.05
	Whittaker(G)	0.35	0.16	0.04
	PSMR(C)	0.39	0.15	0.11
	Whittaker(C)	0.44	0.19	0.06

Taiwan has a well-established national health insurance system; however, significant geographical disparities in life expectancy and mortality rates persist among its counties and cities. The proposed method in the paper allows for identifying townships/districts with homogeneous mortality rates based on population needs by selecting the appropriate number of clusters in K-means. These areas can then be merged into a reference population. By applying a graduation method, estimation errors are reduced, facilitating more stable and reliable life expectancy estimates for small populations.

5. Applications

This study employs the SMR as the primary indicator and utilizes machine learning techniques to classify Taiwan's 368 townships into groups, with similar characteristics within each group and

different characteristics between groups. The Lee (2003) heterogeneity index serves as the criterion for selecting the reference population for each township. By integrating the proposed approach with smoothing methods, we address the challenge of zero age-specific mortality (or zero claims) in small areas and significantly improve the goodness-of-fit of mortality estimates. The construction of age-specific mortality curves for individual products poses significant challenges due to the fact that the annuity illustrated in Figure 14 may still be in the accumulation phase and lacks the necessary statistical information.

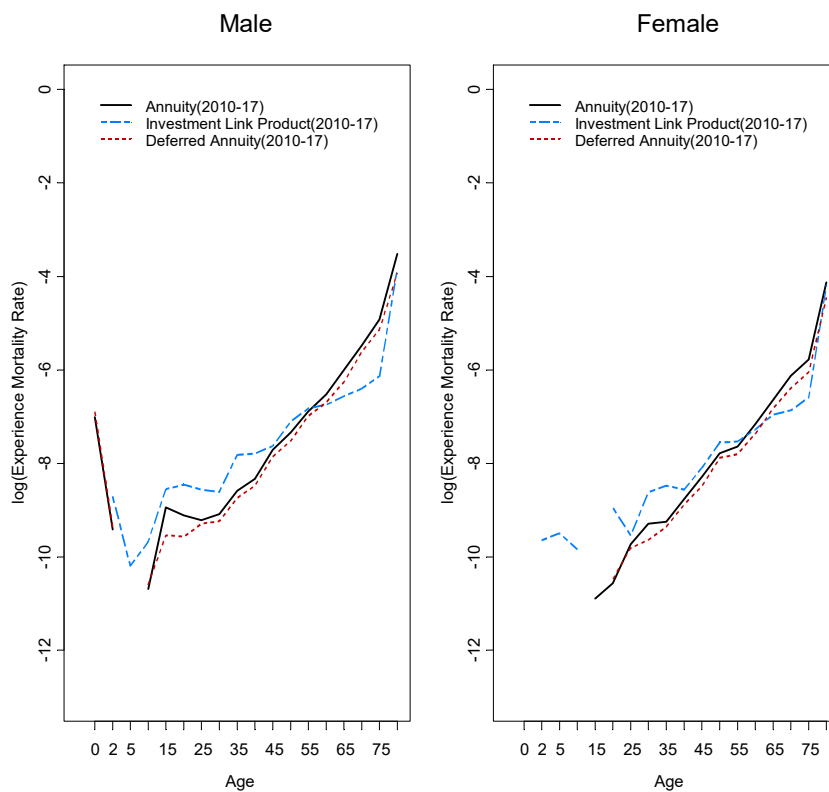


Figure 14. Life Insurance Industry Experience Rates(2010-17)

We classify 16 types of insurance products according to their life, death, and coverage period characteristics (Source: Taiwan Insurance Institute) as follows: Pure Endowment (Term), Pure Endowment (Whole Life), Life Insurance (Term), Life Insurance (Whole Life), Endowment (Term), Endowment (Whole Life), Interest-Sensitive Pure Endowment, Interest-Sensitive Life Insurance (Term), Interest-Sensitive Life Insurance (Whole Life), Interest-Sensitive Endowment (Term),

Interest-Sensitive Endowment (Whole Life), Investment-Linked Product Pure Endowment, Investment-Linked Product Life Insurance (Term), Investment-Linked Product Life Insurance (Whole Life), Investment-Linked Product Endowment (Term), and Investment-Linked Product Endowment (Whole Life). We then computed the annual SMR for the period 2011–2016, using all life insurance policies as the reference population. To identify the optimal number of product groups, PCA was employed. Based on the scree plot and the cumulative explained variance of the principal components, which exceeded 90%, three to four distinct groups were identified. By examining the eigenvectors, which represent the coefficients of the original variables in the linear combinations, the relationships between the principal components and the original product characteristics can be elucidated. As shown in Table 7, the first group, characterized by positive eigenvector loadings, primarily comprises Pure Endowment (Whole Life), Interest-Sensitive Pure Endowment, and Interest-Sensitive Whole Life Insurance products. This clustering pattern suggests a strong association with “savings-oriented” features.

The second group, which also exhibits positive eigenvector loadings, mainly includes Investment-Linked Pure Endowment, Term Pure Endowment, and related products, indicating a predominant association with “investment-oriented” characteristics. The third group, featuring strong loadings on variables linked to term life and investment-linked insurance products, appears to emphasize “protection” as the primary product attribute. Subsequently, the K-means clustering algorithm was applied to further classify the products into four distinct groups. As presented in [Table 8](#), Group 2 predominantly consists of Interest-Sensitive Endowment (Term) and Interest-Sensitive Endowment (Whole Life), reflecting a strong emphasis on “savings” features. Group 1 mainly includes Term Life and Whole Life insurance products, clearly indicating “protection” as the primary product function. Meanwhile, Group 4 is primarily composed of Investment-Linked Whole Life and Investment-Linked Term Endowment products, suggesting a dominant focus on “investment-oriented” characteristics.

In the final stage of the analysis, we calculated the average empirical mortality rates for each product group using data from 2012 to 2016. The results, illustrated in Figure 15, reveal a consistent

pattern for both males and females: Group I (protection) exhibits the highest mortality level, followed by Group II (investment), and Group III (savings), which shows the lowest mortality. This ordering aligns well with established characteristics of the Taiwanese insurance market, where protection-oriented products typically attract higher-risk insureds, while savings-oriented products tend to be purchased by individuals with lower mortality risk.

Table 7. PCA Classification-SMR (Male)

	First	Second	Third
Pure Endowment(Term)	-0.195006634	0.419124131	0.08713258
Pure Endowment(Whole Life)	0.30011353	-0.184729647	-0.10117208
Life Insurance(Term)	-0.288571704	-0.23870062	0.12491127
Life Insurance(Whole Life)	-0.167696046	-0.376000664	-0.23759419
Endowment(Term)	-0.323778922	-0.006925607	-0.07845679
Endowment(Whole Life)	-0.256730516	-0.332458637	-0.09161275
Interest-Sensitive Pure Endowment	0.326294266	-0.06746131	-0.00299632
Interest-Sensitive Life Insurance(Term)	-0.237078211	-0.208917785	0.39524437
Interest-Sensitive Life Insurance(Whole Life)	0.279834391	-0.216401761	0.10070668
Interest-Sensitive Endowment(Term)	-0.326592708	0.05848461	0.05743391
Interest-Sensitive Endowment(Whole Life)	-0.163762398	-0.42303711	0.24117951
Investment Link Product Pure Endowment	-0.046277005	0.2470674	0.59349699
Investment Link Product Life Insurance(Term)	0.164556233	-0.17986622	0.42157407
Investment Link Product Life Insurance(Whole Life)	-0.261179892	0.064733867	-0.36709724
Investment Link Product Endowment(Term)	0.273677041	0.086276987	-0.03558687
Investment Link Product Endowment(Whole Life)	-0.207289013	0.314391697	0.04176866

Table 8. Kmeans Clustering-SMR (Male)

	k=2	k=3	k=4
Pure Endowment(Term)	2	3	2
Pure Endowment(Whole Life)	2	2	4
Life Insurance(Term)	2	2	1
Life Insurance(Whole Life)	2	2	1
Endowment(Term)	2	3	4
Endowment(Whole Life)	2	2	1
Interest-Sensitive Pure Endowment	2	3	2
Interest-Sensitive Life Insurance(Term)---- Outliers	1	1	3
Interest-Sensitive Life Insurance(Whole Life)	2	3	2
Interest-Sensitive Endowment(Term)	2	3	2
Interest-Sensitive Endowment(Whole Life)	2	3	2
Investment Link Product Pure Endowment	2	3	2
Investment Link Product Life Insurance(Term)	2	2	1
Investment Link Product Life Insurance(Whole Life)	2	2	4
Investment Link Product Endowment(Term)	2	3	4
Investment Link Product Endowment(Whole Life)	2	3	2

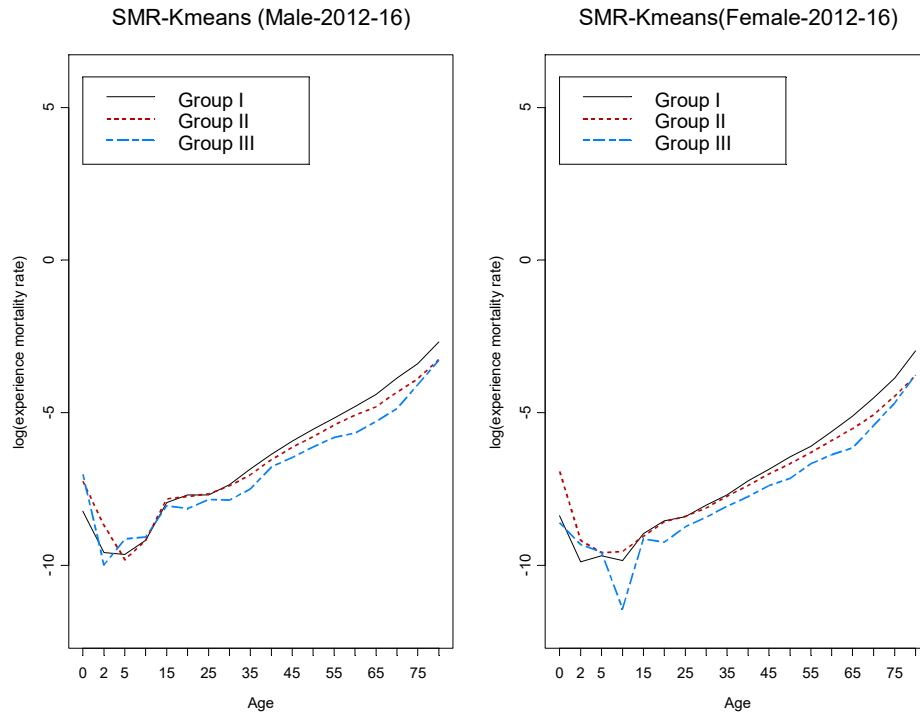


Figure 15. Empirical mortality rates after Kmeans clustering

By examining temporal trends in SMR and incorporating product-type characteristics, we adopted a two-stage analytical framework consisting of PCA and K-means clustering. This approach enabled a robust classification of insurance products into three mortality-risk groups: high, medium, and low. The resulting segmentation corresponds to three conceptual risk categories—protection (mortality risk), savings (survival risk), and investment (longevity risk). This framework facilitates the construction of group-specific empirical life tables, offering practical support for insurers in product design, underwriting, and pricing. Furthermore, by assigning products with limited exposure or zero deaths to a broader reference group, the proposed method effectively alleviates instability arising from insufficient sample size.

6. Conclusion

Taiwan entered an *aging society* ($\geq 7\%$ of the population aged 65 years and older) in 1993 and became a *super-aged society* ($\geq 20\%$ aged 65+) in 2025. The rapid demographic transition poses critical challenges, including labor shortages, inadequate retirement security, economic stagnation,

and increased healthcare expenditure. However, the speed and extent of population aging vary considerably across regions in Taiwan, resulting in place-based differences in needs and resource allocation. For example, the Ministry of the Interior's abridged life tables indicate that the life expectancy gap between Taipei City and Taitung County exceeded seven years in 2023, reflecting substantial disparities in mortality and health outcomes. Moreover, the 2023 Household Income and Expenditure Survey (Directorate General of Budget, Accounting, and Statistics) showed that the income gap between the highest and lowest quintiles surpassed sixfold. These regional disparities underscore the necessity for localized aging policies and the development of region-specific strategies tailored to the health needs of older residents.

In response to corporate risk management requirements, volatility in market interest rates, and the evolving needs of policyholders, life insurers in Taiwan have introduced a diverse range of products over the past decade. In addition to traditional life insurance products, investment-linked policies that integrate financial protection with asset accumulation have become dominant in the market. Despite continuous improvements in mortality, whole-life insurance remains highly favored by Taiwanese policyholders. However, this trend has also emerged as a potential source of financial risk for insurers. For instance, extended life expectancy may result in underestimation of policy reserves for annuity products, thereby increasing insurers' long-term liability exposure.

Accordingly, establishing an appropriate product classification system aligned with differentiated mortality tables is crucial for risk assessment and capital adequacy. The findings of this study suggest that the proposed methodology allows insurers to adopt a risk-based product classification approach. This framework enables more refined mortality risk evaluation and enhances effective management of the overall product portfolio.

Reference

- Ahcan, A., Medved, D., Olivieri, A., and Pitacco, E. (2014). Forecasting Mortality for Small Populations by Mixing Mortality Data. *Insurance: Mathematics and Economics*, 54: 12-27.
- Alexander, M., Zagheni, E., and Magali Barbieri, M. (2017). A Flexible Bayesian Model for

Estimating Subnational Mortality. *Demography*, 54(6): 2025-2041.

- Anson, J. (2018). Estimating Local Mortality Tables for Small Areas: An Application using Belgian Sub-arrondissements. *Quetelet Journal*, 6(1):73–97.
- Baltrus, P., Malhotra, K., Rust, G., Levine, R., Li, C., and Gaglioti, A.H. (2019). Identifying County-Level All-cause Mortality Rate Trajectories and Their Spatial Distribution across the United States. *Preventing chronic disease*, 16: E55.
- Benamins, M.R., Silva, A., Nazia S., Saiyed, and Fernando G. De Maio. (2021) Comparison of all-cause mortality rates and inequities between black and white populations across the 30 most populous US cities. *JAMA Network Open* 4(1): e2032086-e2032086.
- Booth, H., and Tickle, L. (2008). Mortality Modelling and Forecasting: A Review of Methods. *Annals of Actuarial Science*, 3: 3-43.
- Bravo, J.M., and Malta, J. (2010). Estimating Life Expectancy in Small Population Areas. Conference of European Statisticians.
- Brønnum-Hansen, H., Baadsgaard, M. (2012). Widening Social Inequality in Life Expectancy in Denmark. A Register-based Study on Social Composition and Mortality Trends for the Danish Population. *BMC Public Health*, 12: 994.
- Brouhns, N., Denuit, M., and Vermunt, J. K. (2002). A Poisson Log-bilinear Regression Approach to the Construction of Projected Lifetables. *Insurance: Mathematics and Economics* 31 (3), 373–393.
- Cairns, A. J. G., Blake, D., and Dowd, K. (2006). A Two-Factor Model for Stochastic Mortality with Parameter Uncertainty: Theory and Calibration. *Journal of Risk and Insurance*, 73(4): 687-718.
- Cairns, A. J. G., Blake, D., Dowd, K., Coughlan, G. D., Epstein, D., Ong, A., and Balevich, I. (2009). A Quantitative Comparison of Stochastic Mortality Models using Data from England and Wales and the United States. *North American Actuarial Journal*, 13(1): 1-35.
- Cairns, A. J., Blake, D., Dowd, K., Coughlan, G. D., and Khalaf-Allah, M. (2011). Bayesian Stochastic Mortality Modelling for Two Populations. *ASTIN Bulletin: The Journal of the IAA*, 41(1): 29-59.
- Cairns, A.J.G., Kallestrup-Lamb, M., Rosenskjold, C., Blake, D. and Dowd, K. (2019). Modelling Socio-economic Differences in Mortality using a New Affluence Index. *ASTIN Bulletin*, 49(3): 555–590.
- Chen, L., Cairns, A.J.G., and Kleinow, T. (2017). Small Population Bias and Sampling Effects in Stochastic Mortality Modelling. *European Actuarial Journal*, 7(1): 193-230.

- Coulomb, J.B., Salhi, Y., and Thérond, P.E. (2020). Credibility Adjustment of the Lee-Carter Longevity Model for Multiple Populations. Technical report, Hal Open Science.
- D'Amato, V., Haberman, S., Piscopo, G., Russolillo, M. and Trapani, L. (2014). Detecting Common Longevity Trends by a Multiple Population Approach. *North American Actuarial Journal*, 18(1): 139-149.
- Dowd, Kevin, Andrew JG Cairns, David Blake, Guy D. Coughlan, and Marwa Khalaf-Allah. (2011) A gravity model of mortality rates for two related populations. *North American Actuarial Journal* 15(2): 334-356.
- Dwyer-Lindgren, L., Bertozzi-Villa, A., Stubbs, R.W., Morozoff, C., Kutz, M.J., Huynh, C., and Murray, C. J. 2016. US County-level Trends in Mortality Rates for Major Causes of Death, 1980-2014. *The Journal of the American Medical Association* 316(22): 2385-2401.
- Edwards, R.D., and Tuljapurkar, S. (2005). Inequality in Life Spans and a New Perspective on Mortality Convergence Across Industrialized Countries, *Population and Development Review*, 31(4): 645-674.
- Enchev, V., Kleinow, T., and Cairns, A.J.G., (2017). Multi-population Mortality Models: Fitting, Forecasting and Comparisons. *Scandinavian Actuarial Journal*. 4: 319-342.
- Eriksson, C. (2020). Multi-population Mortality Models in the Lee-Carter Framework: An Empirical Evaluation on Sweden's 21 counties. 39.
- Haraty, R.A., Dimishkieh, M., Masud, M. (2015). An Enhanced k-Means Clustering Algorithm for Pattern Discovery in Healthcare Data. *International Journal of Distributed Sensor Networks*. 11(6).
- Hyndman, R.J., Booth, H. and Yasmeen, F., (2013) Coherent Mortality Forecasting: The Product-Ratio Method with Functional Time Series Models. *Demography*, 50(1): 261-283.
- Jarner, S.F., and Kryger, E.M. (2011). Modelling Adult Mortality in Small Populations: the SAINT Model. *ASTIN Bull* 41(02): 377–418.
- Keyfitz, N., and Caswell, H. (2005). *Applied Mathematical Demography*. Springer.
- Lee, R. D., and L. R. Carter. (1992) Modeling and Forecasting U.S. mortality. *Journal of the American Statistical Association*, 87(419): 659-671.
- Lee, W.C. (2003). A partial SMR Approach to Smoothing Age-specific Rates. *Annals of epidemiology* 13(2): 89-99.
- Li, J. S. H., Zhou, R., and Hardy, M. (2015). A Step-by-step Guide to Building Two-population Stochastic Mortality Models. *Insurance: Mathematics and Economics*, 63: 121-134.

- Li, N., and Lee, R. (2005) Coherent Mortality Forecasts for a Group of Populations: An Extension of the Lee-Carter Method. *Demography*, 42(3): 575-594.
- London, R.L. (1985). Graduation: The Revision of Estimates. New Hartford, CT: ACTEX.
- Mitchell, D., Brockett, P., Mendoza-Arriaga, R., and Muthuraman, K. (2013). Modeling and Forecasting Mortality Rates. *Insurance: Mathematics and Economics* 52(2), 275-285.
- Menzietti, M., Morabito, M. F., and Stranges, M. (2019). Mortality Projections for Small Populations: An Application to the Maltese Elderly. *Risks*, 7(2), 35.
- Newell, C. (1988). Methods and Models in Demography, Chichester, Wiley.
- Orwat-Acedańska, A. (2024). Accuracy of Small Area Mortality Prediction Methods: Evidence from Poland. *Journal of Population Research*, 41(1): 1-20.
- Plat, R. (2009) On Stochastic Mortality Modeling. *Insurance: Mathematics and Economics*, 45(3): 393-404.
- Petrelli, A., Ventura, M., Di Napoli, A., Pappagallo, M., Simeoni, S., and Frova, L. (2024). Socioeconomic Inequalities in Avoidable Mortality in Italy: Results from a Nationwide Longitudinal Cohort. *BMC Public Health*. 24(1):757.
- Renshaw, A. E. and S. Haberman. (2006) A Cohort-Based Extension to the Lee-Carter Model for Mortality Reduction Factors. *Insurance: Mathematics and Economics* 38(3), 556-570.
- Singh, G.K., Lin C.C. (2019). Area Deprivation and Inequalities in Health and Health Care Outcomes. *Ann Intern Med* 171(2):131-132.
- Sanzenbacher, G.T., Webb, A., Cosgrove, C. M., and Orlova, N. (2019). Rising Inequality in Life Expectancy by Socioeconomic Status. *North American Actuarial Journal*, 25(sup1): S566–S581.
- Tetzlaff, F., Sauerberg, M., Grigoriev, P., Tetzlaff, J., Mühlichen, M., Baumert, J., Michalski, N., Wengler, A., Nowossadeck, E., and Hoebel, J. (2024). Age-specific and Cause-specific Mortality Contributions to the Socioeconomic Gap in Life Expectancy in Germany, 2003-21: an Ecological Study. *Lancet Public Health*. 9(5): e295-e305.
- Villegas, A. M., Haberman, S., Kaishev, V. K., and Millossovich, P. (2017). A Comparative Study of Two-population Models for the Assessment of Basis Risk in Longevity Hedges. *ASTIN Bulletin: The Journal of the IAA*, 47(3): 631-679.
- Villegas, A. M., and Haberman, S. (2014). On the Modeling and Forecasting of Socioeconomic Mortality Differentials: An Application to Deprivation and Mortality in England. *North American Actuarial Journal*, 18(1): 168-193.
- Wang, H.C., Jin, S., and J. C. Yue. (2012) A Simulation Study of Small Area Mortality Projection.

Journal of Population Studies, 45, 77–110. (In Chinese).

Wang, H. C., C. J. Yue, and C. T. Chong. (2018) Mortality Models and Longevity Risk for Small Populations. *Insurance Mathematics and Economics* 78(1): 351-359.

Yang, S. S., J. C. Yue, and H. C. Huang. (2010) Modeling Longevity Risks Using a Principal Component Approach: A Comparison with Existing Stochastic Mortality Models. *Insurance: Mathematics and Economics* 46(1), 254-270.

Yue, Jack C., Wang, H.C., and Wang, T.Y. (2021). Using Graduation to Modify the Estimation of Lee–Carter Model for Small Populations, *North American Actuarial Journal*, 25:sup1, S410-S420.